Slope Fields, Differential Equations... and Maybe Euler's Method, Too

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Outline

Two Problems

Solving a DE for AP

Three AP Problems

Five Suggestions

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The First Problem

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Possible Answer: Introduce slope fields and Euler's method before differential equations

An Aside

By the way. . .
If we are supposed to approach topics from the *algebraical*, *numerical*, and *graphical* points of view, then shouldn't every AB class learn Euler's method?

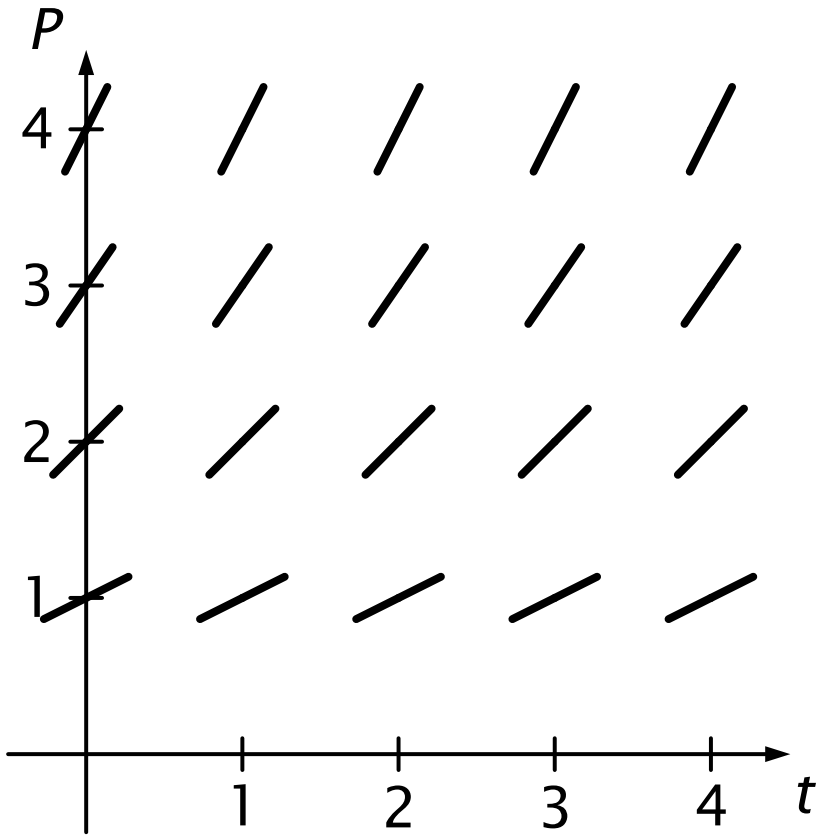
The Second Problem

A population P grows according to $P' = 0.5P$, where $P = 1$ at time $t = 0$. Estimate the value of P at time $t = 3$.

The Slope Field Answer

Given $P' = 0.5P$, where $P(0) = 1$, estimate $P(3)$.

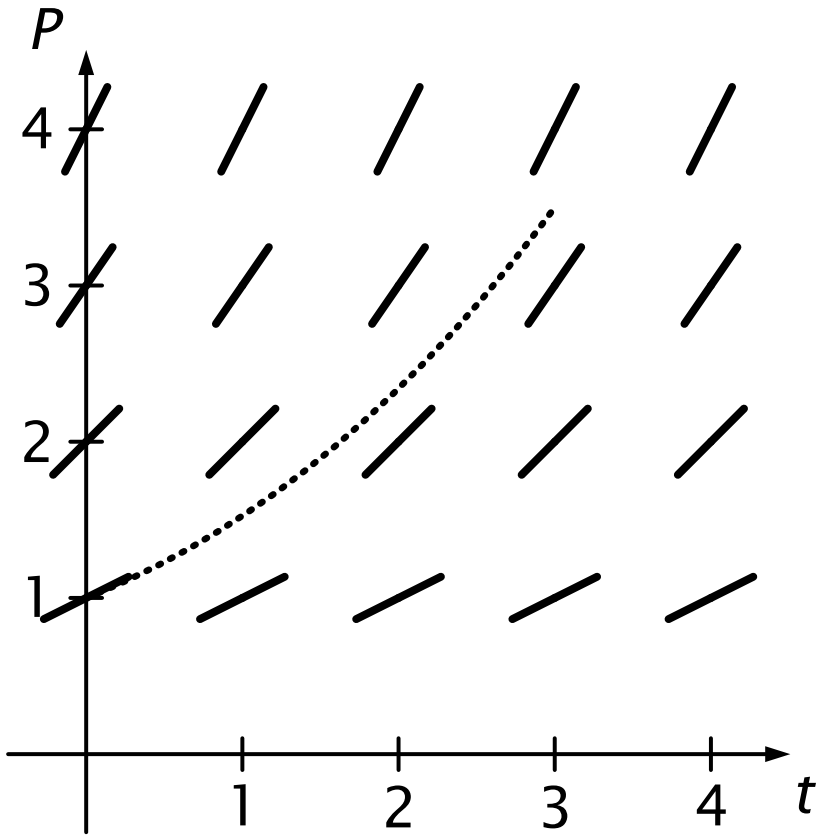
For $P = 1$, $P' = 1/2$
For $P = 2$, $P' = 1$
For $P = 3$, $P' = 3/2$
For $P = 4$, $P' = 2$



The Slope Field Answer

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For $P = 4$, $P' = 2$



The Euler's Method Answer

Euler's method on $y' = f(x, y)$ and initial value (x_0, y_0)

- Given a point (a, b), write the tangent line:

$$\text{Tangent} = y'(a, b)(x - a) + b$$

- Evaluate the tangent at the next x-value to get the next y-value.
- Repeat

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Hence,

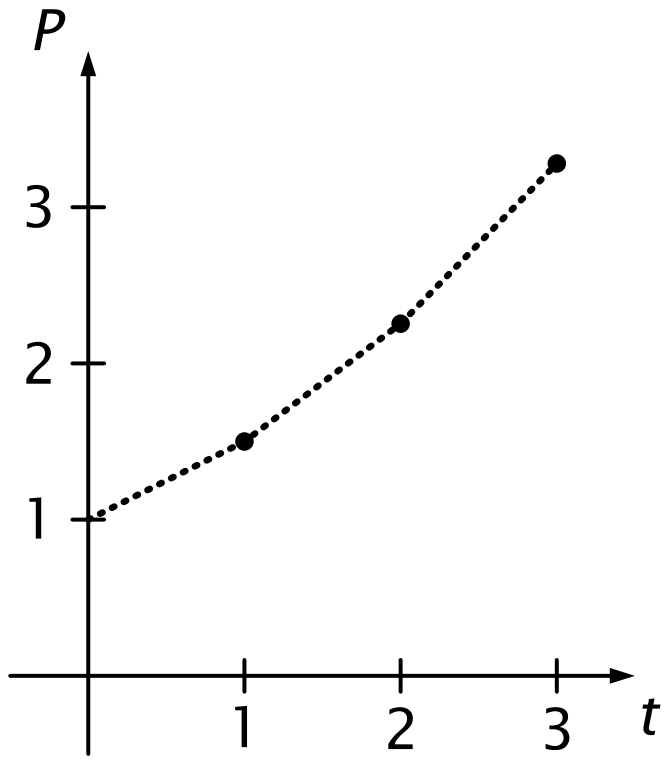
$$\begin{aligned} y_1 &= y'(x_0, y_0)\Delta x + y_0 \\ y_2 &= y'(x_1, y_1)\Delta x + y_1 \\ &\vdots \\ y_n &= y'(x_{n-1}, y_{n-1})\Delta x + y_{n-1} \end{aligned}$$

The Euler's Method Answer

Given $P' = 0.5P$, where $P(0) = 1$, estimate $P(3)$.

We use a change in t of 1; or,
$\Delta t = 1$

$$P(0) = 1$$
$$P(1) = P'(0) + P(0) = 1.5$$
$$P(2) = P'(1) + P(1) = 2.25$$
$$P(3) = P'(2) + P(2) = 3.375$$



The Euler's Method Answer on a TI-84

Given $P' = 0.5P$, where $P(0) = 1$, estimate $P(3)$.

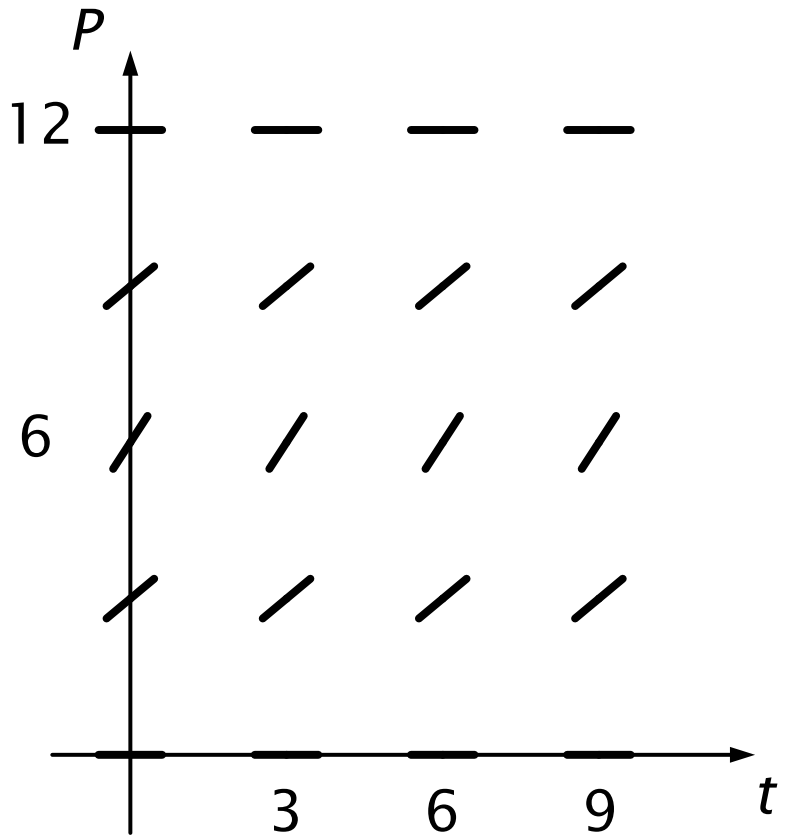
1. Store the initial value: $1 \to P$, press enter
2. Type $0.5P + P \to P$ and press enter
3. Continue to press enter the correct number of times

$1 \to P$	
	1
$0.5P + P \to P$	
	1.5
	2.25
	3.375

A Variation on the Second Problem

A constrained growth model: $P' = \frac{1}{24}P(12 - P)$
(Also called a *logistic* model)

- Max population M is *carrying capacity*; here, $M = 12$
- Population value at which population is growing fastest is $M/2 = 6$
- Connection to AP Environmental Science



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Solving a Differential Equation by AP Standards

Students are expected to

- have the algebra and integration skills needed
- use the initial value
- consider domain issues in the solution
- handle DEs which use variables other than x and y

Solving a Differential Equation by AP Standards

Solve $P' = 0.5P$ with $P(0) = 1$ to find $P(3)$

$$\frac{dP}{dt} = 0.5P$$

$$\int \frac{dP}{P} = \int 0.5\ dt \qquad \text{Separate}$$

$$\ln|P| = 0.5t + C \qquad \text{Antiderivative and constant}$$

$$P = Be^{0.5t} \qquad \text{Solve for dependent variable}$$

$$P(0) = 1 \implies P(t) = e^{0.5t} \qquad \text{Uses initial value}$$

$$P(3) = e^{3/2} \qquad \text{Answer}$$

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Three AP Calculus Free-Response Problems

1. 2004 BC #5 (parts c and d are AB)
2. 2008 AB #5
3. 2014 AB #6

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Suggestions for Your Students

- Practice DEs with variables in every position:

$$x\frac{dy}{dx} = y \qquad y\frac{dy}{dx} = x \qquad \frac{dy}{dx} = xy \qquad xy\frac{dy}{dx} = 1$$

- Do not use x and y all the time; reinforce relationship between dependent quantity and independent variable
- Explain why $\frac{dy}{dx} = x + y$ cannot be solved in AP; but slope fields and Euler's method are expected
- Always check the domain of the solution to a DE
- Reading directions helps!

Thank You!

Have you heard of GA^{2}PMT? That's

The GEORGIA ASSOCIATION of
ADVANCED PLACEMENT MATH TEACHERS!

Annual meeting on the Friday of the Georgia Math Conference at Rock Eagle